# Molecular and electronic structure of 1-Naphtol : Ab initio molecular orbital and density functional study

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*Abstract*— The molecular vibrations of 1-Naphtol were investigated in polycrystalline sample, at room temperature, by FT- IR and FT-Raman spectroscopy. In parallel, ab initio and various density functional (DFT) methods were used to determine the geometrical, energetic and vibrational characteristics of 1-Naphtol . On the basis of B3LYP/6-31G\* and B3LYP/6-311+G\*\* methods and basis set combinations, a normal mode analysis was performed to assign the various fundamental frequencies according to the total energy distribution (TED). The vibrational spectra were interpreted, with the aid of normal coordinate analysis based on a scaled quantum mechanical force field. The Infrared and Raman spectra were also predicted from the calculated intensities. Comparison of simulated spectra with the experimental spectra provides important information about the ability of the computational method to describe the vibrational modes. Simulation of Infrared and Raman spectra, utilizing the results of these calculations led to excellent overall agreement with observed spectral patterns. The investigation is performed using quantum chemical calculations conducted by means of the Gaussian 98W and Guassview set of programs. Further, density functional theory (DFT) combined with quantum chemical calculations to determine the first-order hyperpolarizability.

Keywords- Vibrational spectra; Fourier Transform infrared and FT-Raman spectra; DFT calculation, first-order hyperpolarizability

# I. INTRODUCTION

Accurate vibrational assignment for aromatic and another conjugated system is necessary for characterization of materials. Assignment for complex systems can be proposed on the basis of frequency agreement between the computed harmonics and the observed fundamentals. Quantum chemical computational methods have proven to be an essential tool for interpreting and predicting vibrational spectra [1-2]. A significant advance in this area was made by scaled quantum mechanical (SQM) force field method [3-6]. In the SQM approach the systematic errors of the computed harmonic force field are corrected by a few scale factors which were found to be well transferable between chemically related molecules [2,7-9]. Recent spectroscopic studies on these materials have been motivated because the vibrational spectra are very useful for the understanding of specific biological process and in the analysis of relatively complex systems. In the present study, we extend a probing into the application of the B3LYP/6-31G\* (small basis set) and 3LYP/6-311+G\*\* (large basis set) based on SQM method [2] to vibrational analysis and conformational stability of 1-Naphtol.

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The main difficulty in such investigation is that the vibrational spectra of these compounds have not been completely analyzed until now and generally only rough assignments are available.

The geometrical parameters of the most optimized geometry obtained via energy calculations were used for the DFT calculations. The infrared and Raman intensities were also predicted. Based on these calculations, the simulated FT-IR and FT-Raman spectra were obtained. The observed and the simulated spectra agrees well.

# II. EXPERIMENTAL

The fine polycrystalline samples of 1-Naphtol were obtained from the Lancaster chemical company, UK, and used as such for the spectral measurements. The room temperature Fourier transform infrared spectra of the title compounds were measured in the 4000–400  $\text{cm}^{-1}$  region at a resolution  $\pm 1$  cm<sup>-1</sup> using KBr pellets on Perkin-Elmer RX1 FT-IR spectrophotometer equipped with He-Ne laser source, KBr beam splitter and LiTaO3 detector. Boxcar apodisation was used for the 250 averaged interferograms collected for both the samples and background. The FT-Raman spectra of 1-Naphtol were recorded on a BRUKERIFS-66V model interferometer equipped with an FRA106 and a FT-Raman accessory. The spectra were recorded in the 3500-100 cm<sup>-</sup> stokes region using the 1064 nm line of a Nd:YAG laser for the excitation operating at 200mW power. The reported wave numbers are believed to be accurate within  $\pm 1 \text{ cm}^{-1}$ .

## **III. COMPUTATIONAL DETAILS**

The calculation of the vibrational frequencies is essential and also useful for the vibrational assignments of the spectra. Quantum chemical calculations for 1-Naphtol was performed with the Gaussian 98W program [10] using the Becke 3-Lee-Yang-Parr (B3LYP) functional [11,12] supplemented with the standard B3LYP/6-31G\* (small basis set) and 3LYP/6-311+G\*\* (large basis set) for the Cartesian representation of the theoretical force constants have been computed at the fully optimized geometry by assuming Cs point group symmetry. Scaling of the force field was performed according to the SQM procedure [13,14] using selective (multiple) scaling in the natural internal coordinate representation[15,16]. Transformations of the force field and the subsequent normal coordinate analysis including the least squares refinement of the scaling factors, calculation of total energy distribution (TED) and IR and Raman intensities were done on a PC with the MOLVIB program (Version V7.0-G77) written by Sundius[17,18]. The TED elements provide a measure of each internal coordinates contribution to the normal coordinate. For the plots of simulated IR and Raman spectra, pure Lorentzian band shapes were used with a bandwidth of 10 cm<sup>-1</sup>. The prediction of Raman intensities was carried out by following the procedure outlined below. The Raman activities calculated by the Gaussian 98W program and adjusted during scaling procedure with MOLVIB were converted to relative Raman intensities using the following relationship derived from the basic theory of Raman scattering [19-21].

$$I_{i} = \frac{f(v_{o} - v_{i})^{4} S_{i}}{v_{i} [1 - \exp(-hcv_{i} / kT)]}$$

where  $v_o$  is the exciting frequency (in cm<sup>-1</sup> units),  $v_i$  the vibrational wavenumber of the *i*<sup>th</sup> normal mode, *h*, *c* and *k* are the universal constants and *f* is the suitably chosen common scaling factor for all the peak intensities.

# **IV. PREPARE YOUR PAPER BEFORE STYLING**

The nonlinear response of an isolated molecule in an electric field  $E_i(\omega)$  can be represented as a Taylor expansion of the total dipole moment  $\mu_t$  induced by the field:

$$\mu_{t} = \mu_{0} + \alpha_{ii}E_{i} + \beta_{iik}E_{i}E_{i} + \dots$$

Where  $\alpha$  is linear polarizability,  $\mu_0$  the permanent dipole moment and  $\beta_{ijk}$  are the first-order hyperpolarizability tensor components.

The components of first-order hyperpolarizability can be determined using the relation

$$\beta_{i} = \beta_{iii} + \frac{1}{3} \sum_{i \neq j} (\beta_{ijj} + \beta_{jij} + \beta_{jji})$$

Using the x, y and z components the magnitude of the total static dipole moment ( $\mu$ ), isotropic polarizability ( $\alpha_0$ ), first-order hyperpolarizability ( $\beta_{total}$ ) tensor, can be calculated by the following equations:

$$\mu_1^{0} = (\mu_x^{2} + \mu_y^{2} + \mu_z^{2})^{1/2}$$
  
$$\beta_{\text{tot}} = (\beta_x^{2} + \beta_y^{2} + \beta_z^{2})^{1/2}$$

The complete equation for calculating the first-order hyperpolarizability from Gaussian 98W output is given as follows [10]:

$$\beta_{\text{tot}} = [(\beta_{\text{xxx}} + \beta_{\text{xyy}} + \beta_{\text{xzz}})^2 + (\beta_{\text{yyy}} + \beta_{\text{yzz}} + \beta_{\text{yxx}})^2 + (\beta_{\text{zzz}} + \beta_{\text{zxx}} + \beta_{\text{zyy}})^2$$

The  $\beta$  components of GAUSSIAN 98W output are reported in atomic units, the calculated values have to be converted into electrostatic units (1 a.u= 8.3693 x10<sup>-33</sup> esu)..

Before calculating the hyperpolarizability for the investigated compound, the optimization has been carried out in the UHF (unrestricted open-shell Hartree-Fock) level. Molecular geometries were fully optimized by Berny's optimization algorithm using redundant internal coordinates. All optimized structures were confirmed to be minimum energy conformations. An optimization is complete when it has converged. i.e., when it has reached a minimum on the potential energy surface, thereby predicting the equilibrium structures of the molecules. This criterion is very important in geometry optimization. The inclusion of d polarization and double zeta function in the split valence basis set is expected to produce a marked improvement in the calculated geometry [22]. At the optimized structure, no imaginary frequency modes were obtained proving that a true minimum on the potential energy surface was found. The electric dipole moment and dispersion free first-order hyperpolarizibility were calculated using finite field method. The finite field method offers a straight forward approach to the calculation of hyperpolarizabilities [23]. All the calculations were carried out at the DFT level using the three-parameter hybrid density functional B3LYP and a 3-21 G (d, p) basis set.

## V. RESULTS AND DISCUSSION

# A.Molecular geometry

The optimized molecular structure of 1-Naphtol was shown in Fig.1. The global minimum energy obtained by the DFT structure optimization was presented in Table 1.

Detailed description of vibrational modes can be given by means of normal coordinate analysis (NCA). For this purpose, the full set of 72 standard internal coordinates containing 21 redundancies were defined. From these, a nonredundant set of local symmetry coordinates were constructed by suitable linear combinations of internal coordinates following the recommendations of Fogarasi et. al [15, 16] are summarized. The theoretically calculated DFT force fields were transformed in this later set of vibrational coordinates and used in all subsequent calculations.

## B. Analysis of vibrations spectra

The 51 normal modes of 1-Naphtol are distributed among the symmetry species as  $\Gamma_{3N-6}$ = 35 A' (in-plane) + 16 A" (out-of-plane), and in agreement with C<sub>s</sub> symmetry. All the vibrations were active both in Raman scattering and infrared absorption. In the Raman spectrum the in-plane vibrations (A') give rise to polarized bands while the out-of-plane ones (A") to depolarized band. The TED were reported. For visual comparison, the observed and simulated FT-IR and FT-Raman spectra of 1-Naphtol are produced in a common frequency scales in Fig. 2 & Fig. 3. Root mean square (RMS) values of frequencies were obtained in the study using the following expression,

RMS = 
$$\sqrt{\frac{1}{n-1}\sum_{i}^{n} |v_{i}^{calc} - v_{i}^{exp}|^{2}}$$

The RMS error of the observed and calculated frequencies (unscaled / B3LYP/6-311+G\*\*) of 1-Naphtol was found to be 107 cm<sup>-1</sup>. This is quite obvious; since the frequencies calculated on the basis of quantum mechanical force fields usually differ appreciably from observed frequencies. This is partly due to the neglect of an harmonicity and partly due to the approximate nature of the quantum mechanical methods. In order to reduce the overall deviation between the unscaled and observed fundamental frequencies, scale factors were applied in the normal coordinate analysis and the subsequent least square fit refinement algorithm resulted into a very close agreement between the observed fundamentals and the scaled frequencies. Refinement of the scaling factors applied in this study achieved a weighted mean deviation of 9 cm<sup>-1</sup> between the experimental and scaled frequencies of the title compound.

## **C-C vibrations:**

The bands between 1480 and 1650 cm<sup>-1</sup> are assigned to C-C stretching modes [24]. In the present study, the carbon stretching vibrations of the title compound have been observed at 1572,1636 cm<sup>-1</sup> in the FT-IR and 1572 cm<sup>-1</sup> in FT-Raman spectrum. These assignments are in good agreement with literature [25,26]. In present investigation, the C-C mode mixes with C–H in-plane bending vibrations. **C-H vibrations:** 

The presence of hetero-aromatic-type structure is best recognized by the presence of C–H stretching vibrations [27] near 3030 cm<sup>-1</sup>. Aromatic compounds commonly exhibit multiple weak bands in the region 3100–3000 cm<sup>-1</sup> due to aromatic C–H stretching vibrations. The bands due to C–H in-plane ring bending vibration interacting with C–C stretching vibration are observed as a number of m-w intensity sharp bands in the region 1300–1000 cm<sup>-1</sup>. C–H out-of-plane bending vibrations are strongly coupled vibrations and occur in the region 900–667 cm<sup>-1</sup>. Accordingly, in the present study the C–H vibrations of the title compounds are observed at 3177, 3186 cm<sup>-1</sup> in the FT-IR spectrum and 3166 in FT-Raman for 1-Naphtol .

## **C-O vibrations:**

The non-linearity of hydrogen bond in 1-Naphtol have an impact over the carbonyl group frequency. The interaction of carbonyl group with the other group present in the system does not produce such a drastic and characteristic changes in the frequency of C O stretch. The carbonyl stretching frequency is very sensitive to the factors that disturb the nature of the carbonyl group and its precise frequency is characteristic of the type of the carbonyl compound being studied. Particularly detailed correlations have been made for the carbonyl bond stretching frequency. The carbonyl stretching frequency has been most extensively studied by infrared spectroscopy. This multiply bonded group is highly polar (> $C^{\delta_{+}}=O^{\delta_{-}}$ ) and therefore gives rise to an intense infrared absorption band. The carbon-oxygen double bond is formed by the  $p_{\pi}$ - $p_{\pi}$  bonding between carbon and oxygen.

Because of the different electro-negativities of carbon and oxygen atoms, the bonding electrons are not equally distributed between the two atoms. The following two resonance forms contribute to the bonding of the carbonyl group >C=O  $\leftrightarrow$  C<sup>+</sup>-O<sup>-</sup>. The lone pair of electrons on oxygen also determines the nature of the carbonyl group. The position of the C=O stretching vibration is very sensitive to various factors such as the physical state, electronic effects by substituents, ring strains, etc. [24]. Consideration of these factors provides further information about the environment of the C=O group. The carbonyl stretching generally occurs as a strong absorption in the region from 1730 to 1645  $\text{cm}^{-1}$ . This portion of the infrared spectrum is most useful because the position of the carbonyl absorption is quite sensitive to substitution effects and the geometry of the molecule. Accordingly, in the present investigation, the peaks identified at 1810 cm<sup>-1</sup> have been assigned to C=O stretching vibrations for 1-Naphtol.

## **O-H vibrations**

The precise positions of O–H band are dependent on the strength of hydrogen bond. The O–H stretching appears at  $3590-3400 \text{ cm}^{-1}$  in the inter-molecular hydrogen bonded systems. The observed peaks in this region are sharp and strong. The title compounds in this study showed a very strong absorption peak at  $3753 \text{ cm}^{-1}$  which are due to the O–H stretching vibrations.

# C-O stretching and O-H bending vibrations

Two bands arising from C–O stretching and O–H bending appear in the spectra of carboxylic acids near 1210–1320 and 1400–1440 cm<sup>-1</sup>, respectively. Both these bands involve some interaction between C–O stretching and inplane C–O–H bending. The more intense band near 1280–1315 cm<sup>-1</sup> for dimers is generally referred to as CO stretching band and it usually occurs as a doublet in the spectra of long-chain fatty acids. One of the characteristic bands in the spectra of dimeric carboxylic acid arises from the out-of-plane bending of the hydrogen bonded OH. It appears near 1070 cm<sup>-1</sup> and is characteristically broad with medium intensity [24].

## **Ring vibrations:**

Several ring modes are affected by the substitution in the aromatic ring. In the present study, the bands absorbed at 1110, 1176 cm<sup>-1</sup> and 479,585 cm<sup>-1</sup> have been designated to ring in-plane and out-of-plane bending modes, respectively. For most of the remaining ring vibrations, the overall agreement is satisfactory. Small changes in frequencies observed for these modes are due to the changes in force constants/reduced mass ratio resulting mainly due to the extent of mixing between ring and substituent group. VI. HYPERPOLARIZABILITY CALCULATIONS

The first-order hyperpolarizibility  $(\beta_{ijk})$  of the novel molecular system of 1-Naphtol is calculated using 3-21 G (d,p) basis set based on finite field approach. Hyperpolarizibility is a third rank tensor that can be described by a 3 x 3 x 3 matrix. It strongly depends on the method and basis set used. The 27 components of 3D matrix can be reduced to 10 components due to Kleinman symmetry. The calculated first-order hyperpolarizability  $(\beta_{total})$  of 1-Naphtol is  $1.47325 \times 10^{-30}$  esu, which is nearly seven times that of urea (0.1947 x  $10^{-30}$  esu). The calculated dipole moment ( $\mu$ ) and first-order hyperpolarizibility ( $\beta$ ) are shown in Table 2. The theoretical calculation seems to be more helpful in determination of particular components of  $\beta$ tensor than in establishing the real values of  $\beta$ . Domination of particular components indicates on a substantial delocalization of charges in those directions. It is noticed that in  $\beta_{xxx}$  (which is the principal dipole moment axis and it is parallel to the charge transfer axis) direction, the biggest values of hyperpolarizability are noticed and subsequently delocalization of electron cloud is more in that direction. The higher dipole moment values are associated, in general, with even larger projection of  $\beta_{total}$  quantities. The electric dipoles may enhance, oppose or at least bring the dipoles out of the required net alignment necessary for NLO properties such as  $\beta_{total}$  values. The connection between the electric dipole moments of an organic molecule having donor-acceptor substituent and first hyperpolarizability is widely recognized in the literature [27]. The maximum  $\beta$  was due to the behavior of non-zero  $\mu$  value. One of the conclusions obtained from this work is that non-zero µ value may enable the finding of a non-zero  $\beta$  value. Of course Hartee–Fock calculations depend on the mathematical method and basis set used for a polyatomic molecule.

### VII. CONCLUSION

In this work, the SQM force field method based on DFT calculations at the B3LYP/6-311+G\*\* level have been carried out to analyze the vibrational frequencies of 1-Naphtol . Refinement of the scaling factors applied in this study achieved a weighted RMS deviation of 9 cm<sup>-1</sup> between the experimental and scaled frequencies. This close agreement established between the experimental and scaled frequencies obtained using large basis set (6-311+G\*\*) calculations has proved to be more reliable and accurate than the calculations using lower basis sets. The first-order hyperpolarizability ( $\beta_{iik}$ ) of the novel molecular system of 1-Naphtol is calculated using 3-21 G (d,p) basis set based on approach. calculated first-order finite field The hyperpolarizability ( $\beta_{total}$ ) of 1-Naphtol is 1.47325x10<sup>-30</sup> esu, which is nearly seven times that of urea (0.1947 x  $10^{-30}$  esu). References

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Figures and Tables



Fig. 1. The optimized molecular structure of 1-Naphtol

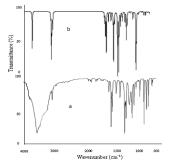


Fig. 2 FT-IR spectra of 1-Naphtol . (a) Observed (b) Calculated with B3LYP/6-311+G\*\*

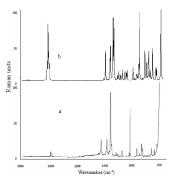


Fig. 3 FT-Raman spectra of 1-Naphtol . (a) Observed (b) Calculated with B3LYP/6-311+G\*\*

Table 1. Total energies of 1-Naphtol , calculated at DFT

Method	<b>Energies</b> (Hartrees)			
6-31G*	-461.084753			
6-311+G**	-461.10918545			
(B3LYP)/6-31G* and (B3LYP)/6-311+G** level				

Table 2. The	dipole	mo	ment	(μ)	and	first-order
hyperpolariz	ibility	(β)	of 1-1	Naph	tol de	rived from
DFT calculations						

DFT calculations					
β <sub>xxx</sub>	24.6798306				
β <sub>xxy</sub>	4.1106034				
β <sub>xyy</sub>	2.7371947				
β <sub>yyy</sub>	-3.5868028				
β <sub>zxx</sub>	-118.5181275				
β <sub>xyz</sub>	-0.7739399				
β <sub>zyy</sub>	1.3003588				
$\beta_{xzz}$	-12.4290667				
β <sub>yzz</sub>	5.030306				
β <sub>zzz</sub>	-36.030306				
$\beta_{total}$	1.289537				
μ <sub>x</sub>	2.1196756				
μ <sub>y</sub>	0.0228399				
μ <sub>z</sub>	0.570441				
μ	2.195211				

Dipole moment (µ) in Debye, hyperpolarizibility  $\beta(-2\varpi; \omega, \omega) = 10^{-30}$ esu.